Journal of Aircraft

VOLUME 6 MAY-JUNE 1969 NUMBER 3

Sonic Boom Theory

R. SEEBASS
Cornell University, Ithaca, N. Y.

I. Introduction

THE theory of the phenomenon we call the sonic boom is not a complex one; it is simply the consistent first-order theory of the flowfield about an aircraft moving supersonically through a stratified atmosphere. The region of the flowfield which is of primary interest in this theory extends from several, to several hundred, aircraft lengths. The main complication is that our most elementary theory, which is a linearized one, does not provide a consistent first-order description of the flowfield far from the aircraft. Let's recall what we mean by first order. Any aircraft, as it moves through the air, produces disturbances in physical variables whose ratio with their undisturbed values is proportional to the ratio of some measure of the thickness of the aircraft to its length. For practical aircraft this "slenderness ratio" is always small. A typical value for SST aircraft is 1:20. When we speak of a first-order theory, we mean one that computes departures from the undisturbed flow which are the same size as, that is, the same order as, the slenderness ratio of the aircraft. Second-order quantities are roughly the same size as the square of this ratio.

As was already mentioned, our simplest theory—the linearized one—is a first-order theory that is not consistently correct to this order as we proceed farther and farther from the aircraft. To construct a consistent first-order theory we need to correct this linearized theory for cumulative second-order effects that eventually make a first-order contribution.

The first part of this paper outlines the basic theory of sonic booms for the case of steady flight in an atmosphere without winds. If the reader accepts two or three results that are not completely obvious, then with a little fluid mechanics, trigonometry, and elementary calculus, the theory can be deduced rather simply. Unsteady flight, steady winds, and atmospheric turbulence introduce effects which, although most of them can be accounted for, are beyond the scope of this paper.

In developing this simple theory we concentrate on certain general conclusions one may draw from it, so that in Sec. IV we can examine briefly the feasibility of reducing the impact of the sonic boom by aerodynamic means.

II. Sonic Boom Theory

Let's begin our whirlwind tour of sonic boom theory. There are two basic approaches to such a theory; the easiest route to the results we desire draws upon both approaches. They are illustrated in Fig. 1. In the aerodynamic approach we assume the aircraft to be fixed in a steady supersonic stream; in the acoustic approach, we consider the aircraft moving in a medium at rest. The latter approach is the easiest to extend to unsteady aircraft motions. In either approach the radial and azimuthal coordinates r and θ are convenient ones. The acoustic time coordinate t multiplied by the aircraft speed U is analogous to the x coordinate of the aerodynamic approach. The Mach angle and the Prandt-Glauert factor β are defined in the figure. The ray tube shown emanating from the aircraft in the acoustic approach is defined by the normals to the acoustic wavefronts. The distance along the ray tube will be denoted by s, whereas the phase variable ξ is measured from the location of the pressure disturbance that is generated by

Richard Seebass received his BSE (1958) and MSE degrees from Princeton, and the PhD degree from Cornell University in 1962. He has remained with Cornell since receiving his degree and is currently an Associate Professor of Aerospace Engineering and Acting Director of the Center for Applied Mathematics. He spent a year (1966–1967) at NASA Headquarters on a contract basis through Cornell University. While there, he was a staff member of the Research Division of the Office for Advanced Research and Technology and contributed to the establishment of the current NASA contract program on generation and propagation of sonic booms. At present he is a consultant to the National Academy of Sciences Committee on the SST-Sonic Boom, the Federal Aviation Administration's Office of Noise Abatement, and an Advisor to the Sonic Boom Research Panel of the Interagency Aircraft Noise Abatement Program. He has been a contributor to other areas of fluid mechanics research. He is a member of the AIAA.

Presented at the Annual Aviation and Space Division Conference of the American Society of Mechanical Engineers, June 19, 1968. Received November 8, 1968; revision received December 2, 1968. The preparation of this survey was sponsored by NASA Grant NGR-33-010-054.

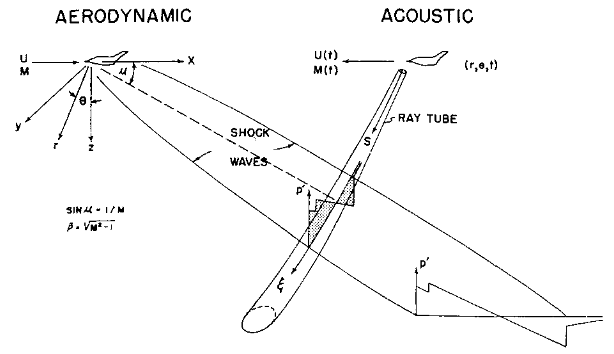

Fig. 1 Aerodynamic and acoustic points of view.

the passage of the aircraft. Aside from a scale factor, the pressure signature in the two approaches is the same.

There are three distinct regions in both approaches: 1) a local region near the aircraft where the flow is fully three-dimensional, 2) the mid-field region centered about Mach waves in which the distance away from the Mach waves is small compared to the distance away from the aircraft, and 3) the far-field region where the pressure signature of the aircraft has acquired the shape that persists to infinity.

It is convenient to examine the mid-field region, which is the region of primary interest to us, in three steps. First we consider the linearized aerodynamic theory for the mid-field region. Next we put ourselves in a coordinate system moving with the wave down a ray tube and momentarily neglect ray tube area changes and atmospheric effects. In this coordinate system we correct the linearized theory for cumulative second-order effects. Finally, we use the concepts of geometrical acoustics to adjust these results so that they account for ray tube area changes and atmospheric stratification.

Figure 2 depicts the linearized theory of the mid-field region. Consider the pressure field at x,r,θ. The disturbance here can only result from disturbances that originated in the fore Mach cone of this point. Indeed, one can show that if $x - \beta r \ll r$, then the local pressure disturbance is composed of two parts. One is due to the cross-sectional area of the aircraft as cut by the tangent to the fore Mach cone; the other is due to the component of the force, on the perimeter of the cross-section, which is perpendicular to the freestream and lies in the θ = const plane. That is,

$$p_o'(x,r;\theta) = p_V'(x,r;\theta) + p_L'(x,r;\theta)$$

where p_V' is the pressure disturbance generated by a slender body of revolution whose cross-sectional area distribution $S_V(x;\,\theta)$ is the same as the area cut by the fore Mach cone projected on to an x = const plane, and where p_L' is the pressure field generated by a body of revolution with the cross-sectional area distribution

$$S_L(x;\,\theta) = \beta \int_0^x \mathcal{L}(\tilde{x};\theta) d\tilde{x}/\rho U^2$$

Here $\mathcal{L}$ is the component of force perpendicular to the freestream that lies in the θ = const plane. Note that as $x \to \infty$,

$$S_L(x;\theta) \to \beta(\text{lift}) \cos\theta/\rho U^2$$

Thus, we see that pressure disturbance for a complex three-dimensional configuration can be reduced to the calculation of the pressure field due to an equivalent body of revolution $S(x;\theta) = S_V(x;\theta) + S_L(x;\theta)$, provided that $x - \beta r \ll r$. That is,

$$\frac{p_o'(x,r;\theta)}{\gamma p M^2} \cong \frac{1}{(2\beta r)^{1/2} 2\pi} \int_0^{x-\beta r} \frac{S''(\tilde{x};\theta)}{(x - \beta r - \tilde{x})^{1/2}} d\tilde{x}$$

The function

$$F(t;\theta) = \frac{1}{2\pi} \int_0^t \frac{S''(x;\theta)}{(t - x)^{1/2}} dx$$

which was first introduced by Whitham,[1] is often a more convenient one than the pressure. This function, usually referred to as the Whitham function, has the following useful and easily verified properties:

$$\int_0^\infty F(t\,\theta)\,dt = 0; \qquad F(t;0) \sim \frac{\beta \text{lift}}{4\pi\rho U^2 t^{3/2}} \text{ as } t \to \infty$$

Throughout we will use $(\ )'$ to indicate disturbance quantities and $(\ )_o$ to distinguish linearized disturbance values from the corrected quasi-linear ones. As mentioned previously, this linearized theory fails far from the body. Indeed, it does not provide shock waves across which physical quantities are discontinuous, but only Mach surfaces across which the derivatives of physical quantities are discontinuous. To illustrate this failure and explain the appropriate remedy, let us switch from the steady aerodynamic approach to the acoustic point of view. As long as we are only interested in second-order effects, we may assume the flow is isentropic. Any shock waves that appear in our theory will only cause third-order changes in the entropy. Also, for the moment let us consider the flow to be one-dimensional and in a steady, uniform medium. Then, if we consider small-amplitude waves of only one family, that is a flow in which all the perturbations travel in the same direction, one can show that the second-order equation for the perturbation velocity is the inviscid form of Burger's equation. That is,

$$\frac{\partial v'}{\partial t} + \left(a + \frac{\gamma - 1}{2} v' + v'\right)\frac{\partial v'}{\partial s} \equiv \frac{\partial v'}{\partial t} + (a + \Gamma v')\frac{\partial v'}{\partial s} = 0$$

Here a is the sound speed in the undisturbed medium and γ the ratio of the specific heats. Now acoustics, or the linearized theory, replaces the exact convection term which consists of the local sound speed plus the local convective velocity with a. We need to examine the effect of the next-order term $\Gamma v'$ that results from a change in the sound speed (due to the pressure disturbance) of $[(\gamma - 1)/2]v'$, and from the convection v'. If we change from the coordinate system s,t to a coordinate system following the linearized disturbance, $\xi = s - at$, $\tau = t/\Gamma$, then the equation becomes

$$\partial v'/\partial\tau + v'(\partial v'/\partial\xi) = Dv'/D\tau = 0$$

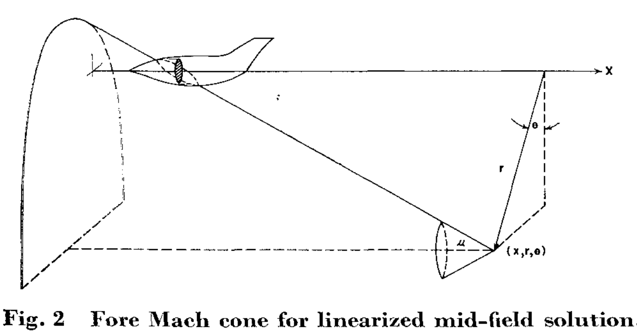

Fig. 2 Fore Mach cone for linearized mid-field solution.

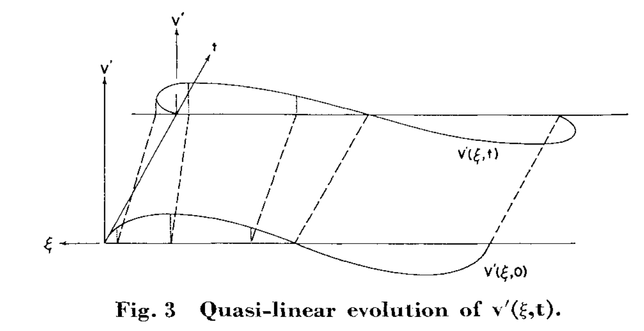

Fig. 3 Quasi-linear evolution of v'(ξ,t).

Thus, values of v' are preserved along particle paths in the new coordinate system. As a consequence, if our initial disturbance was $v'(\xi,0)$, as shown in Fig. 3, then at a later time it will have evolved to $v'(\xi,t)$ as shown. It is clear that there is a first-order distortion in wave shape for large times. Unfortunately, this construction leads to a solution which is triple-valued and untenable on physical grounds. Naturally, this occurs because we have not included viscosity, which prevents $\partial v'/\partial\xi$ from ever becoming infinite.

Introducing viscosity to prevent the solution from becoming triple-valued is, in the limit of vanishing viscosity, the same as introducing a shock wave that renders the solution discontinuous but otherwise single-valued. There are numerous equivalent methods of introducing shock waves. One way that requires no previous information is to integrate the equation from $\xi = -\infty$ to $\xi = +\infty$, and assume that as $\xi \rightarrow \pm\infty$, $v'(\xi,t) \rightarrow 0$. A better procedure uses the fact that a weak shock wave bisects the characteristics that run into it. For simplicity we adopt the first procedure, which gives

$$\int_{-\infty}^{\infty} \frac{\partial v'}{\partial t}\, d\xi = \frac{d}{dt}\int_{-\infty}^{\infty} v'\, d\xi = -\Gamma \int_{-\infty}^{\infty} v' \frac{\partial v'}{\partial \xi}\, d\xi = 0$$

or

$$\int_{-\infty}^{\infty} v'\, d\xi = \text{const}$$

This simple rule tells us to locate the shock waves in such a way that we preserve the area under the $v'(\xi)$ curve, making the solution discontinuous but otherwise single-valued. This procedure allows us to calculate the decay of the pressure pulse once a shock wave has formed. Consider the triangular profile of amplitude v_o' and wavelength λ_o shown in Fig. 4. In our linear theory this profile would not change with time. But our quasi-linear theory says that each point on this profile will advance a distance proportional to its amplitude. The advance of a point with amplitude v_o' is

$$\alpha = \Gamma \int_0^t v_o'(\tilde{t})\, d\tilde{t} = \Gamma \int_0^s \frac{v_o'(\tilde{s})\, d\tilde{s}}{a}$$

We see from the figure that to introduce a discontinuity, i.e., a shock wave, that renders the solution unique, we must have $v_o'\lambda_o = v_s'\lambda$. We can also note from the figure that

$$v_o'/v_s' = (\alpha + \lambda_o)/\lambda$$

Thus, the amplitude of the discontinuity is given by

$$v_s' = v_o'(1 + \alpha/\lambda_o)^{-1/2}$$

Note that the position of the shock wave is determined by making the two shaded areas equal.

Now that we know how to correct our basic result for cumulative second-order effects by allowing the wave shape to distort and by introducing shock waves, we need only provide the rule by which we adjust these results for ray tube area changes and atmospheric inhomogeneities. To do this we need the simple theory of geometrical acoustics. Figure 5 depicts a side and a front view of a ray tube. Recall that the normals

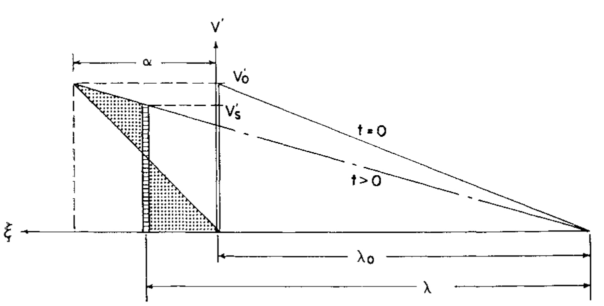

Fig. 4 Diagram showing location of shock wave.

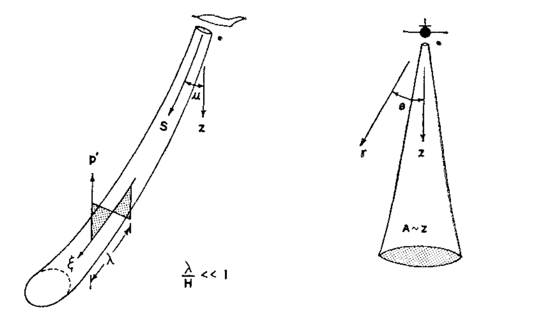

Fig. 5 Ray tubes in geometrical acoustics.

to the acoustic wave fronts define the ray tube. Thus, in an isothermal atmosphere the ray tube area is $\sim r$; directly below the aircraft it is $\sim z$.

If we consider such a ray tube, then as long as the scale H of atmospheric inhomogeneities is large compared to the wavelength of our signature, that is $\lambda/H \ll 1$, then one can show that the reflection and diffraction of energy due to ray tube area changes may be neglected. Consequently the energy flux is constant along the ray tube; this flux is composed of an internal energy flux and a pressure work term. Because the internal energy flux is proportional to the pressure work we can conclude that $p_o'v_o'A$ = const along ray tubes, where A is the area of the ray tube. This result, combined with the simple consequence of linearized theory, $v_o' = p_o'/\rho a$, leads to the condition that

$$p_o'^2 A/\rho a = p_*'^2 A_*/\rho_* a_* = \text{const}$$

along a ray tube. This theory is, of course, a linear one, and we need to correct it for quasi-linear effects. Because

$$p_s'/p_o' = v_s'/v_o = (1 + \alpha/\lambda_o)^{-1/2}$$

and

$$\alpha = \Gamma \int_{s*}^{s} \frac{v_o' d\tilde{s}}{a} = \Gamma \int_{s*}^{s} \frac{p_o'}{\rho a^2}\, d\tilde{s}$$

we may conclude that the pressure behind the shock wave is

$$\frac{p_s'}{p_*'} = \left(\frac{\rho a A_*}{\rho_* a_* A}\right)^{1/2} \left[1 + \frac{\Gamma}{\lambda_o} p_*' \left(\frac{A_*}{\rho_* a_*}\right)^{1/2} \int_{s*}^{s} \frac{d\tilde{s}}{(\rho a^3 A)^{1/2}}\right]^{-1/2}$$

Here $(\ )_*$ refers to reference conditions that are to be determined by the linear theory.*

In general, the quadrature in the preceding formula must be carried out numerically. However, for an isothermal atmosphere in which $\rho = \rho_* \exp(z - z_*)/H$, the numerical results are given by the error function. If we consider the $\theta = 0$ plane, then below the aircraft the pressure behind the shock is

$$\frac{p_s'}{p_*'} = \left(\frac{p_* z}{p z_*}\right)^{1/2} \left\{1 + \frac{\Gamma M}{\beta\lambda_o} \frac{p_*'}{\gamma p_*} (2\pi H z)^{1/2} \left[\operatorname{erf}\left(\frac{z}{2H}\right)^{1/2} - \operatorname{erf}\left(\frac{z_*}{2H}\right)^{1/2}\right]\right\}^{1/2}$$

When $H \rightarrow \infty$, we obtain the result that applied for all z in a homogeneous atmosphere:

$$\frac{p_s'}{p_*'} = \left(\frac{z_*}{z}\right)^{1/2} \left[1 + \frac{\Gamma M}{\beta\lambda_o} \frac{p_*'}{\gamma p_*} 2(z z_*)^{1/2}\right]^{-1/2}$$

Thus, in a homogeneous atmosphere the asymptotic decay of p_s' is $p_s' \sim z^{-3/4}$. However, in an isothermal atmosphere,

---

* The reference pressure[2] p_*' is related to the Whitham F function by $p_*'(t;\theta)/\rho_* a_*^2 = MF(t;\theta)/(2\beta A_*)^{1/2}$.

Table 1 Typical local and the average atmospheric scale heights

Altitude, ft	H_{av}, ft	H_{local}, ft
20,000	32,000	29,500
40,000	29,000	22,000
60,000	25,000	21,000
80,000	23,000	21,000

if $z/2H \gg 1$, then below the aircraft

$$\frac{p_s'}{p_*'} = \left(\frac{pz_*}{p_*z}\right)^{1/2}\left[1 + \frac{\Gamma M}{\beta\lambda_o}\frac{p_*}{\gamma p_*}2(z_*)^{1/2}\left(\frac{\pi H}{2}\right)^{1/2}\right]^{-1/2}$$

and we see that the asymptotic behavior is $(p_*/p)^{1/2}\, p_s' \sim z^{-1/2}$. This decreased decay rate results when the wavelength of the signal has reached its asymptotic value. In a homogeneous atmosphere the wavelength continues to grow as $z^{1/4}$.

III. General Consequences

The extension of this theory to give the full pressure signature, not just p_s', is simple indeed. We compute the advance of each point relative to points with zero advance. Either this advances the point so far that the solution becomes triple-valued and the signature is truncated by a shock wave, or it does not. In the latter case we must compute the distorted signature shape. For a nonidealized atmosphere this must be done numerically. However, this can be done without specifying the signature shape. We simply need tabulated values of

$$\int_{s_*}^{s}\frac{d\tilde{s}}{[\rho(\tilde{s})a^3(\tilde{s})A(\tilde{s})]^{1/2}}$$

along various ray paths, the simplest and most important path being the one directly under the aircraft, and the ray paths themselves.

In the stratosphere the atmosphere is essentially isothermal and the density decay nearly exponential. Thus, the error function result is quite satisfactory so long as it is used in the stratosphere. See Table 1 for typical local and average atmospheric scale heights.

Plotkin[2] has tabulated the various quantities needed to calculate the signature shape for the International Civil Aviation Organizations' standard atmosphere. His results indicate that the increase in ray tube area due to the temperature gradient in the troposphere compensates to a large degree for the effects of increased density there.

By examining the preceding results for the advance in isothermal and homogeneous atmospheres, we can see that the signature shape that exists at $z = (\pi/2)H$ in a homogeneous atmosphere is the asymptotic shape that occurs below the aircraft in an isothermal atmosphere. This, of course, is a simple consequence of equating the asymptotic advance that occurs in an isothermal atmosphere with the advance that occurs in a homogeneous atmosphere. This important fact was first pointed out by Hayes.[3] The advance that has occurred at $z = (\pi/2)H$ in an isothermal atmosphere is 97.4% the total ultimate advance; at $z = 2H$ it is within 99.5% of its final value.

Although it is possible to conceive of aircraft with overpressure signatures that never evolve to the extent that shock waves occur—and we shall examine this possibility further—this is not the case for present supersonic aircraft. For the typical SST, such as the Boeing 2707, the far-field overpressure signature below the aircraft consists of a shock wave, followed by an expansion that is linear with time after or distance from the passage of the shock wave, to a pressure that is about as far below the ambient pressure as the pressure behind the initial shock wave was above it. This linear decrease is terminated by a second shock wave, which brings the pressure back to near its ambient value. The return to ambient pressure occurs in a weak isentropic compression that is a consequence of the cylindrical nature of the wave propagation involved. This far-field signature is aptly described by the terminology "N-wave."

One way of characterizing this signature is by the pressure rise across the front (and, below the aircraft, the strongest) shock wave; this is often called the overpressure level. For reasons that will be clear later, we shall use the terminology "shock pressure rise." If we take the results of our theory and relate the reference conditions to the actual aircraft parameters, then it is easy to show that as $l/h \to 0$ the components of the shock pressure rise, p_s', have the following behavior:

$$p_s'/P_g \propto \beta^{1/8}V^{1/2}\cos^{1/2}\theta/h^{1/2}l^{3/4}\,(P_a/P_g)^{1/2}$$

$$p_s'/P_g \propto \beta^{3/8}W^{1/2}\cos\theta/Mh^{1/2}l^{1/4}$$

Here h, l, W, and V are the altitude, length, weight, and volume of the aircraft. The reference pressure P_a is the ambient pressure at the flight altitude h and P_g is the ambient pressure at the ground. There are two contributions, of course, one due to volume, the other due to the lift or weight of the aircraft. While the volume contribution decays more rapidly with altitude, the lift contribution decays more rapidly with azimuth angle. Neither contribution depends strongly on the Mach number. The constant of proportionality depends on the shape of the aircraft. For the volume contribution this constant may be zero, as indeed it is for the axisymmetric version of Busemann's bi-plane. For the lift contribution this constant has a minimum value under the proviso that we do not change the Bernoulli constant of the flow. It is easy to see that the higher the altitude the lower the overpressure p_s', and the larger the relative significance of the lift contribution.

The complete theory, including the effects of steady winds, is the result of a large number of individual contributions. Some of the more important ones are listed here along with the year the contribution was made: Landau, 1945; Blokhintsev, 1946; Hayes, 1947[1,4–16], 1954; Friedrichs, 1948; Whitham, 1950, 1952, 1953, 1956; Lighthill, 1954; Keller, 1954; Busemann, 1955; Lomax, 1956; and Guiraud, 1965.

IV. Reducing the Sonic Boom

We now employ the theory just outlined to discuss some of the aerodynamic means by which we may be able to reduce the "impact" of the sonic boom. A number of aerodynamic schemes for reducing the impact of the sonic boom have been proposed in the last few years. Some embody ideas that may eventually prove useful to the designers of real aircraft; others are untenable in that they do not satisfy the basic laws of physics. We have picked two examples from among those that fall in the first category. They are listed below along with various parameters that are directly related to the magnitude of the sonic boom.

Engine streamtube area reduction
Design aircraft with less annoying signatures
. . .
Increase: lift/drag ratio
Decrease: specific fuel consumption, structural weight
Increase: altitude capability, length

The ellipsis indicates that the first part of the list is illustrative and incomplete. Two items on the list relate directly to the over-all efficiency of the aircraft. Any improvement in the efficiency of the aircraft results both in lower operating costs and reduced sonic boom overpressures. We have included these last three items to indicate that we must not overlook the possibility of straightforward improvements in the state of the art in these areas bringing substantial sonic boom overpressure reductions.

As mentioned earlier, we cannot escape the boom due to lift without modifying the Bernoulli constant of the flow.

The equivalent slender body of revolution due to lift never closes; it has a finite base area far downstream that is proportional to the lift. Aircraft engines, however, modify the Bernoulli constant of the flow that passes through them, and it is not entirely unreasonable to think of aircraft in which a reduction in engine streamtube area is used to compensate for the growth of the equivalent body of revolution due to lift. Hypothetically, it is possible to eliminate totally the boom due to lift in this way. This was first pointed out by Resler.[17]

Naturally, the second law of thermodynamics requires that infinitely far downstream the exit streamtube area be larger than the entering one. This pluming of the engine exhaust, which takes many aircraft lengths to occur, gives rise to a weak pressure disturbance at the ground. A portion of this disturbance accounts for the support of the aircraft by the pressure field there. Because the disturbance is spread out over such a large longitudinal distance, it is reasonable to assume that no significant steepening of the disturbance into shock waves occurs. A detailed engine cycle analysis carried out by Resler[18] indicates how very difficult it is to make gains by this route with nominal restrictions on engine size and turbine inlet temperature. Total elimination of the exhaust stream tube area in the present SST configurations would reduce the boom due to lift by about 5%. However, should it eventually prove practical to design aircraft with engines whose streamtube capture area is a significant fraction of $\beta W/\rho U^2$, then reasonable gains may be expected by this route.

We conclude with a discussion of the possibility of designing aircraft with their overpressure signatures modified to reduce human annoyance. Here we tread dangerous ground. Our understanding of which features of the overpressure signature are the most undesirable is far from complete. If shock waves are absent from the signature and the compression rise times are on the order of hundredths of a second, then there is little acoustical energy present in the audible range. The most annoying feature of the sonic boom, as experienced outdoors, lies in the shock waves themselves. On the other hand the low frequencies, which are inaudible outdoors, contain the majority of the acoustical energy and cause structures to vibrate and windows to rattle. As a consequence, even an overpressure signature without shock waves may not be much less annoying indoors than its fully steepened counterpart. Let us leave these unanswered questions with the ad hoc assumption that one of the most annoying features of the sonic boom, at least as it is experienced outdoors, is due to the presence of shock waves in the overpressure signatures. We now turn to the technical and answerable question of whether it is indeed possible to design practical aircraft with overpressure signatures that do not contain shock waves, and if not, then for a given aircraft length and flight conditions, what is the minimum achievable shock pressure rise.

Several years ago F. E. McLean of the NASA Langley Research Center noted that it takes several hundred aircraft lengths for the overpressure signature to reach its asymptotic form.[19] This led him to examine the requirements on aircraft length and weight for a fixed altitude and Mach number to achieve finite rise-time overpressure signatures. The results of his most recent study are shown in Fig. 6.[20]

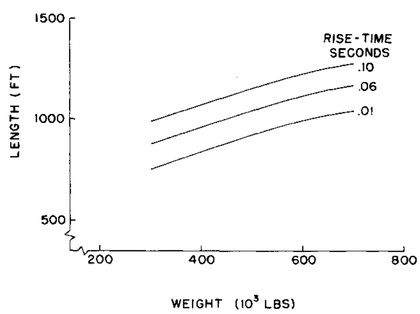

Fig. 6 McLean's results for various finite rise-times; $M = 2.7$, $h = 61{,}000$ ft.

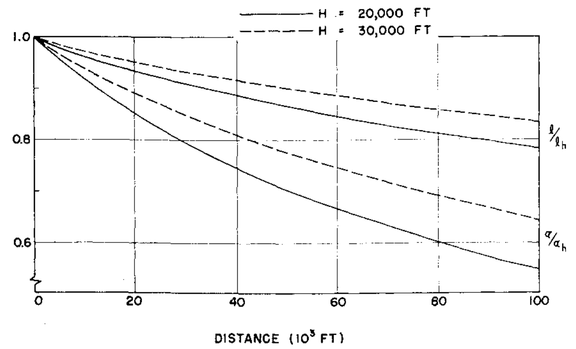

Fig. 7 Ratio of advance and aircraft length for an isothermal atmosphere to their values in a homogeneous atmosphere.

At first glance, these results seems to be fairly pessimistic ones. We see that to achieve a hundredth-of-a-second rise-time, instead of the front and rear shock waves, requires that a 600,000-lb aircraft be nearly 1000 ft long. McLean assumed that the only effect of the atmosphere was that it altered the overpressure level, not the signature shape. But, as Hayes has pointed out, the signature shape that exists $(\pi/2)H$ atmospheric scale heights from the aircraft in a homogeneous atmosphere is the shape that is achieved asymptotically in an isothermal atmosphere. Thus, we can expect further improvement in McLean's results as a result of atmospheric effects.

Figure 7 depicts the ratio of the advance that would occur in an isothermal atmosphere of scale height H, relative to that which would occur in the same distance in a homogeneous atmosphere. Note that, for a typical cruise altitude of 65,000 ft and a scale height appropriate to the stratosphere, the exponentially increasing density has reduced the advance to 65% of the advance that would occur in a uniform medium. Unfortunately, as a simple calculation will show, the aircraft length required is proportional to the $\frac{2}{5}$ power of the advance, and for these conditions would be 84% of the length computed by McLean. The aircraft length ratio, $l/l_h = (\alpha/\alpha_h)^{2/5}$ is also shown in Fig. 7.

An important unanswered question is how much advantage can one take of the isentropic tail pressure wave of the signature to help eliminate the rear shock wave and reduce the aircraft length requirement. Figure 8 shows the pressure signature, in terms of the Whitham function, of an aircraft that has been designed to take some advantage of this tail wave. Here we have matched the asymptotic behavior of the pressure for large x to a linear pressure decrease following the pressure peak. In terms of the Whitham function we have taken the tail wave to be

$$F(x;0) \sim -\beta W/4\pi\gamma p_* M^2(x - \delta)^{3/2}$$

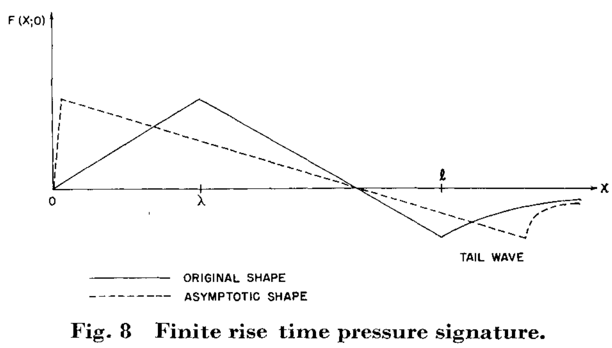

Fig. 8 Finite rise time pressure signature.

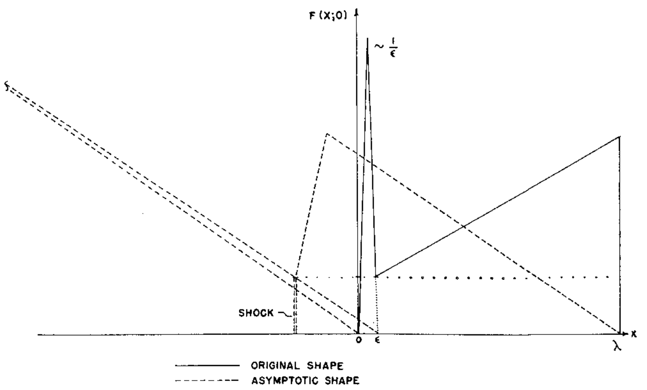

Fig. 9 Minimum shock pressure rise signature.

where δ is an undetermined constant. If we match the pressure at $x = l$ to the tail wave pressure, demand that the pressure gradients be sufficiently small so that no rear shock occurs, require that

$$\int_0^\infty F(t;\theta)\,dt = 0$$

and invert an earlier result to find

$$\frac{\beta W}{\gamma p_* M^2} = 4\int_0^l F(t;0)(l-t)^{1/2}\,dt$$

then we obtain four algebraic equations relating the unknowns l, λ, δ, and W. The solution of these equations gives

$$l = 3.26[(3\beta W/8\pi P_a)(\alpha/\alpha_h)kh^{1/2}]^{2/5}$$

Here α_h is the advance that would occur in a homogeneous atmosphere and $k = (\Gamma M^2/\gamma\beta)(2/\beta)^{1/2}$. If λ^* is the length required to eliminate the front shock wave alone for the same weight W, then for the pressure signature considered here l must be $1.43\lambda^*$ in order to eliminate both shock waves.

Had we considered a pressure signature that was antisymmetrical about $x = \lambda^*$, we would find

$$l = \{2/[4(2)^{1/2} - 5]^{2/5}\}\lambda^* \approx 2.36\lambda^*$$

where

$$\lambda^* = \{\tfrac{15}{16}(\beta W/P_a)(\alpha/\alpha_h)kh^{1/2}\}^{2/5}$$

Thus, we see that the use of the natural tail wave can be quite helpful in eliminating the rear shock wave. Undoubtedly this particular calculation is overly optimistic. To carry out the calculation properly we must use the actual behavior of the tail wave, rather than its asymptotic behavior. These details remain to be worked out. Extensive numerical studies carried out by McLean indicate that $l = 2\lambda$ is not unreasonable, and the factor 2 has been used in determining the lengths shown in Fig. 6.

The point to be made here is that even with the Bernoulli constant of the flow fixed, the question of minimum sonic boom shock pressure rise is a closed one. For any given aircraft weight, flight Mach number and altitude, if the aircraft is sufficiently long, then shock waves may be avoided altogether. This is a consequence of two phenomena: 1) the slow evolution of the signature toward its asymptotic shape in a homogeneous atmosphere and 2) the "freezing" of the signature shape that occurs because of the nearly exponential increase in atmospheric density with distance below the aircraft. While the lengths required for total elimination of the shock waves in the aircraft signature are beyond our present structural capability, this possibility may eventually be realized as improvements in specific fuel consumption, lift to drag ratio and structural weight accrue.

If we assume that we must accept shock waves in our signature, then following the arguments of Jones,[21] it is clear that the signature that will minimize the front shock pressure rise is that indicated in Fig. 9. Here we allow a large pressure peak (essentially a δ function) followed by the linear increase of our finite-rise time signature: to maximize the lift for a given shock pressure rise, the pressure must be as large as possible without adding any further contribution to the shock pressure rise. As a consequence, for a given weight and length aircraft and given flight conditions, there is minimum shock pressure rise.

We are interested in the limit $\epsilon \to 0$ as this leads to the minimum possible shock pressure rise; however, it also leads locally to infinite pressures. Still, we expect the limit $\epsilon \to 0$ to give shock pressure rises that, although they can never be achieved, can actually be approached in practice.

It is a simple matter to apply the results given in our review of the theory to deduce that for an isothermal atmosphere this minimum shock pressure rise is

$$\frac{p_s'}{(P_aP_g)^{1/2}} = \frac{2}{3k}\frac{1}{(2\beta)^{1/2}}\frac{\alpha_h}{\alpha}\,\frac{\lambda}{h}\left\{\left[1+\frac{6}{5}\left(\frac{15}{16}\frac{\beta Wk\alpha h^{1/2}}{P_a\alpha_h\lambda^{5/2}}-1\right)\right]^{1/2}-1\right\}$$

The condition that $p_s' = 0$ recovers our previous result for the length at which there is no front shock wave at all.

To generalize results such as this one for an isothermal atmosphere to a real atmosphere, we simply need to multiply the pressure rise by $[(Ai/A)(Ta/Tg)]^{1/2}$, where Ai/A is the ratio of the isothermal ray tube area to the real ray tube area, evaluated at the ground, and Ta/Tg is the ratio of the temperature of flight altitude to the temperature at the ground.

The maximum pressure in this signature will be larger than the shock pressure rise. If we ask what the minimum achievable overpressure is, then it is clear that the pressure must be constant after the initial pulse and equal to the pressure behind the shock. In this case the result† is

$$\frac{p_{so}'}{(P_aP_g)^{1/2}} = \frac{2}{3k}\frac{1}{(2\beta)^{1/2}}\frac{\alpha_h\lambda}{\alpha\;h}\left\{\left[1+\frac{9}{8}\frac{\beta Wk\alpha h^{1/2}}{P_a\alpha_h\lambda^{5/2}}\right]^{1/2}-1\right\}$$

If we examine the preceding formulas in the limit $\lambda/h \to 0$ then we recover the far-field lower bound of Jones[21] for lift alone. We have not included the volume contribution for two reasons. First, as Busemann pointed out, in principle it can

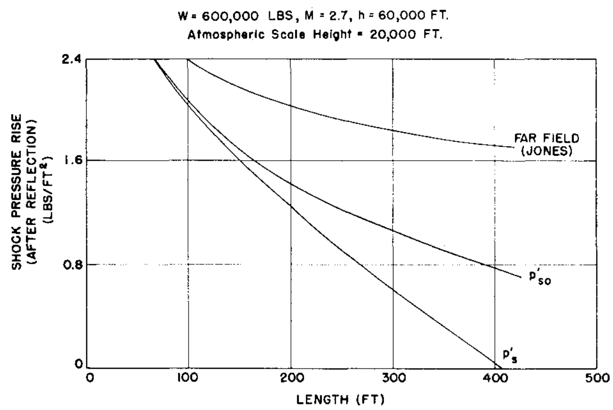

Fig. 10 Front shock pressure rise as a function of aircraft length for the conditions specified. The rear shock has not been attenuated here.

† These results, which generalize the far-field lower bounds given by Jones to the mid-field region, were derived by me (and also by my colleague A. R. George) subsequent to my oral presentation of this material.[23]

be eliminated and need not contribute to the pressure signature; second, for the aircraft and altitudes of interest for commercial supersonic transport, the lift contribution is the dominant one. Because the volume contribution has been eliminated, the foregoing formulas represent the minimum attainable shock pressure rise and overpressure.

As an example of the last two results, we consider a 600,000-lb aircraft flying at a Mach number of 2.7 at 60,000 ft. Figure 10 shows the minimum front shock wave pressure rise and overpressure as a function of aircraft length for these conditions. Also indicated on the figure is the far-field lower bound of Jones. Substantial gains accrue because of midfield effects. If the aircraft is 300 ft long, then the first result gives a front shock pressure rise of 0.62 psf and the second an overpressure of 1.04 psf. Unfortunately the rear-shock wave in both cases would have a significantly larger pressure rise.

It seems that to complete the signature of Fig. 9 in such a way as to minimize the front and rear shock pressure rises simultaneously, we need to rotate the front lobe of the signature clockwise through 180°. Actually the body may be truncated short of this full length. The residual tail wave that exists naturally behind the signature would allow such a truncation without increasing the rear shock pressure rise above that of the front shock wave. Just how much advantage we may take of this fact is not yet clear. For this reason we have not made any attempt to take any advantage of it here. As a consequence, the lengths obtained will not be true optimums.

The proof that this is the optimum signature, ignoring tail wave effects, is a straightforward but a lengthy one. We simply give the results here, noting that true optimum shock pressure rise, including the tail wave effects, must be between these values and those given previously for the front shock wave alone. For the full anti-symmetrical signature of length l, we find[23]

$$\frac{p_s'}{(P_aP_g)^{1/2}} = \frac{2}{3k}\frac{\sigma}{(2\beta)^{1/2}}\frac{\alpha_h}{\alpha}\frac{l}{h}\left\{\left[1+\frac{6\tau}{5\sigma^2}\left(\frac{15}{16\tau}\frac{\beta W k\alpha h^{1/2}}{P_a\alpha_h l^{5/2}}-1\right)\right]^{1/2}-1\right\}$$

and

$$\frac{p_{so}'}{(P_aP_g)^{1/2}} = \frac{2}{3k}\frac{\sigma}{(2\beta)^{1/2}}\frac{\alpha_h}{\alpha}\frac{l}{h}\left\{\left[1+\frac{9}{8\sigma^2}\frac{\beta W k\alpha h^{1/2}}{P_a\alpha_h l^{5/2}}\right]^{1/2}-1\right\}$$

where $\sigma = 1 - 2^{-1/2}$ and $\tau = 1 - [5/4(2)^{1/2}]$. Note that as $l/h \to 0$, these results again reduce to the far-field lower bounds given by Jones. Below the aircraft, the rear shock wave of the far-field signature is always weaker than the front shock wave. Consequently, no additional length is required.

Figure 11 depicts the maximum shock pressure rise (front or rear) as a function of aircraft length for the conditions of

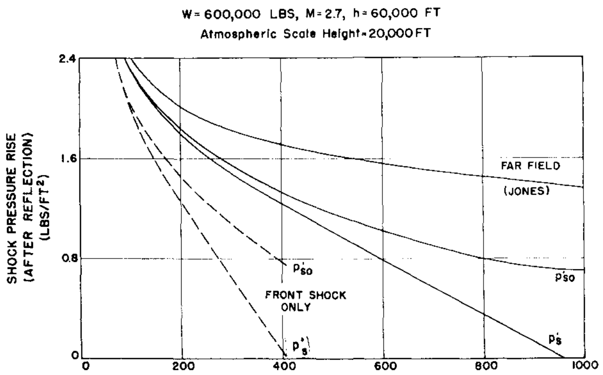

Fig. 11 Shock pressure rise as a function of aircraft length for the conditions specified. The rear shock is weaker than the front for the lengths given here.

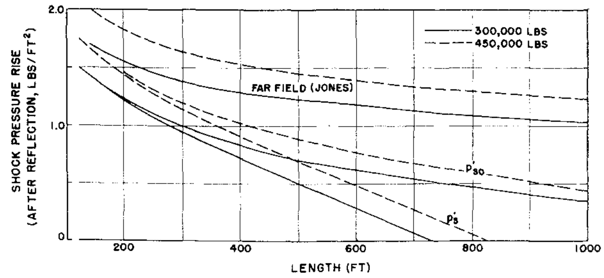

Fig. 12 Shock pressure rise as a function of aircraft length for typical domestic SST weights. Altitude = 60,000 ft; Mach number = 2.7; atmospheric scale height = 20,000 ft.

Fig. 10. Also indicated (by the dashed line) is the length for a given front shock pressure rise, p_s'. The absolute optimum for equal front and rear shock pressure rise must lie between the two results. At this stage, we can only conjecture that it lies nearer the upper curve and hope that we are wrong.

Figure 12 depicts these same results for two weights that are more appropriate to a domestic SST. With the reduced range requirement of domestic operation there is a considerable saving in aircraft weight. Note that a 300-ft, 300,000-lb domestic SST should be able to operate with overpressures below 1.0 psf. Indeed, it seems likely that such levels are achievable even for a 450,000-lb aircraft of the same length. Whether such configurations are practical remains to be determined. Antonio Ferri[24] has studied practical configurations that have pressure signatures that embody some of the principles set forth here. His results are indeed promising and indicate that such overpressure levels are not unrealistic ones.

V. Conclusion

In conclusion, it seems that we can safely prognosticate a continual evolution of SST designs with greatly improved sonic boom characteristics. While major gains may be expected from improvements in the over-all efficiency of such aircraft, as well as through novel design features such as the ones just discussed, we do not foresee any revolutionary concept that will totally eliminate the sonic boom. Recent research efforts seem to be pointing the way to the design of a domestic SST with cruise overpressures of less than 1 psf. Should the shock pressure rise prove to be a more critical parameter than the maximum overpressure level, an even lower value of this quantity seems likely. Whether or not these evolutionary gains will be sufficient to make a domestic SST an economically viable concept is as yet unknown. We cannot hope to provide the answer until we know what features of the overpressure signature are the most annoying ones, and what integrated overpressure loadings are likely to prove to be acceptable.

References

[1] Whitham, G. B., "The Flow Pattern of a Supersonic Projectile," *Communications on Pure and Applied Mathematics*, Vol. 5, 1952, pp. 301–348.

[2] George, A. R. and Plotkin, K., "Sonic Boom Waveforms and Amplitudes in a Real Atmosphere," to be published.

[3] Hayes, W. D., "Brief Review of the Basic Theory," *Sonic Boom Research*, edited by R. Seebass, SP-147, 1967, NASA, pp. 3–7.

[4] Landau, L. D., "On Shock Waves at Large Distances from the Place of Their Origin," *Prikladnaya Matematika Mekhanika*, Vol. 9, 1945, pp. 286–292; also *Journal of Physics*, Academy of Sciences, USSR, Vol. 6, 1942, p. 229.

[5] Blokhintsev, D. I., "The Propagation of Sound in an Inhomogeneous and Moving Medium I," *The Journal of the Acoustical Society of America*, Vol. 18, 1946, pp. 322–328; also "Acoustics of a Nonhomogeneous Moving Medium," Gostekhiazat, Moscow,

USSR, 1946; translated as NACA-TM-1399.

[6] Hayes, W. D., "Linearized Supersonic Flow," thesis, 1947, California Institute of Technology; reprinted as North American Aviation Rept. AL-222; available as Princeton University AMS Report 852.

[7] Hayes, W. D., "Pseudotransonic Similitude and First-Order Wave Structure," *Journal of the Aeronautical Sciences*, Vol. 21, 1954, pp. 721–730.

[8] Friedrichs, K. O., "Formation and Decay of Shock Waves," *Communications on Pure and Applied Mathematics*, Vol. 1, 1948, pp. 211–245.

[9] Whitham, G. B., "The Behavior of Supersonic Flow Past a Body of Revolution Far from the Axis," *Proceedings of the Royal Society* (London), Vol. A201, 1950, pp. 89–109.

[10] Whitham, G. B., "The Propagation of Weak Spherical Shocks in Stars," *Communications on Pure and Applied Mathematics*, Vol. 6, 1953, pp. 397–414.

[11] Whitham, G. B., "On the Propagation of Weak Shock Waves," *Journal of Fluid Mechanics*, Vol. 1, 1956, pp. 290–318.

[12] Lighthill, M. J., "Higher Approximations in Aerodynamic Theory," *General Theory of High Speed Aerodynamics*, edited by W. R. Sears, Sec. E, Vol. 6, pp. 345–489.

[13] Keller, J. B., "Geometrical Acoustics, I, The Theory of Weak Shock Waves," *Journal of Applied Physics*, Vol. 25, 1954, pp. 938–947.

[14] Busemann, A., "On the Relation Between Minimizing Drag and Noise at Supersonic Speeds," *Proceedings of the Conference on High-Speed Aeronautics*, edited by A. Ferri, 1955, pp. 133–144.

[15] Lomax, H., "The Wave Drag of Arbitrary Configurations in Linearized Flows as Determined by Areas and Forces in Oblique Planes," RM A55 A18, 1955, NACA.

[16] Guiraud J. P., "Acoustic géométrique, bruit ballistique des avions supersoniques et focalisation," *Journal de Mecanique*, Vol. 4, 1965, pp. 215–267.

[17] Resler, E. L., Jr., "Reduction of Sonic Boom Attributed to Lift," *Sonic Boom Research*, edited by I. R. Schwartz, SP-180, 1968, NASA, pp. 99–106.

[18] Resler, E. L., Jr., "Lifting Aerodynamic Configurations with No Sonic Boom," *AFOSR-UTIAS Symposium on Aerodynamic Noise*, 1968, to be published.

[19] McLean, F. E., "Some Nonasymptotic Effects on the Sonic Boom of Large Airplanes," TN D-2877, 1965, NASA.

[20] McLean, F. E., "Configuration Design for Specified Pressure Signature Characteristics," *Sonic Boom Research*, edited by I. R. Schwartz, SP-180, 1968, NASA, pp. 37–45.

[21] Jones, L. B., "Lower Bounds for Sonic Bangs." *Journal of the Royal Aeronautical Society*, Vol. 65, 1961, pp. 433–436.

[22] Jones, L. B., "Lower Bounds for Sonic Bangs in the Far Field," *Aeronautical Quarterly*, Vol. 18, 1967, pp. 1–21.

[23] Seebass, R., "Minimum Sonic Boom Shock Strengths and Overpressures," *Nature*, in press.

[24] Ferri, A., "A Report on Sonic Boom Studies," *Sonic Boom Research*, edited by I. R. Schwartz, SP-180, 1968, NASA, pp. 73–98.

MAY-JUNE 1969 J. AIRCRAFT VOL. 6, NO. 3

Study of a Family of Supersonic Inlet Systems

NORMAN E. SORENSEN* AND DONALD B. SMELTZER*
NASA Ames Reasearch Center, Moffett Field, Calif.

AND

ROBERT W. CUBBISON*
NASA Lewis Reasearch Center, Cleveland, Ohio

A theoretical and experimental program is reviewed showing the performance capabilities of a family of three large-scale axisymmetric inlet systems. The major objective of the program was to investigate relatively short inlet systems capable of high performance over the complete Mach number range. The tradeoff of important performance parameters such as engine-face pressure recovery and distortion for boundary-layer bleed was determined. The three inlets were designed for Mach numbers 2.5, 3.0, and 3.5. Vortex generators were used just downstream of the throats to control the flow distortion at the engine face. The Mach number 2.5 and 3.5 designs had engine airflow bypass systems. The results have shown that high performance was more difficult to achieve as the design Mach number increased, because the boundary layer occupied an increasing percentage of the flow in the throat. Also, the bypass systems have not presented serious performance problems.

Nomenclature

D = capture diameter
h = throat height
M = Mach number
m = mass flow
p_p = Pitot pressure
p_t = total pressure
$\bar{p}_t$ = area-weighted average total pressure
s = vortex generator height
α = angle of attack
δ = boundary-layer height

Subscripts

BL = bleed
BP = bypass
DES = design
max = maximum
min = minimum
u = unstart
2 = engine face
∞ = freestream

Presented as Paper 68-580 at the AIAA 4th Propulsion Joint Specialist Conference, Cleveland, Ohio, June 10–14, 1968; submitted June 10, 1968; revision received October 31, 1968.

* Research Scientists.